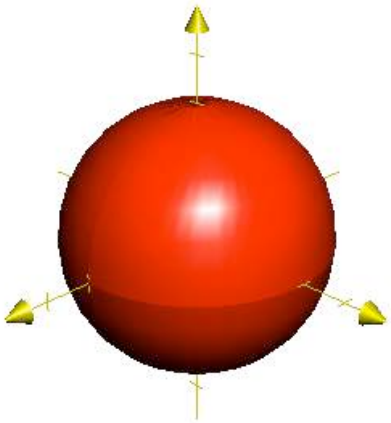
SUPERFICIES CUADRÁTICAS Y FAMOSAS

Geometría del Espacio *Mtra. Jeanett López García*
UNAM-FES Acatlán

1. ESFERA



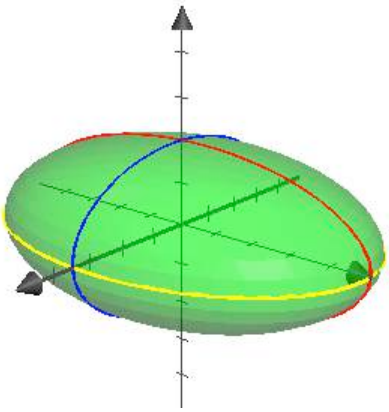
Forma cartesiana

$$x^2 + y^2 + z^2 = r^2$$

Forma Paramétrica

$x = rsenu\cos v$ donde:
$y = rsenu\cos v$ $u \in (0, 2\pi)$
$z = r\cos v$ $v \in (0, \pi)$
r>0

2. ELIPSOIDE



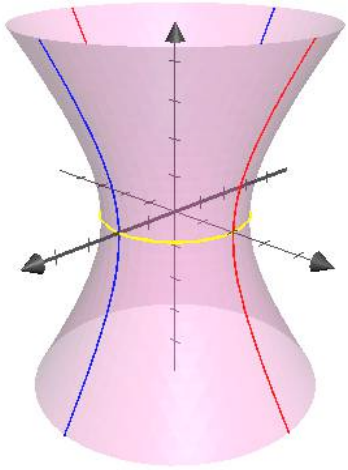
Forma cartesiana

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Forma Paramétrica

$x = asenu\cos v$ donde:
$y = bsenusenv$ $u \in (0, 2\pi)$
$z = c\cos v$ $v \in (0, \pi)$
a,b,c>0

3. HIPERBOLOIDE ELÍPTICO (DE 1 MANTO -HOJA-)



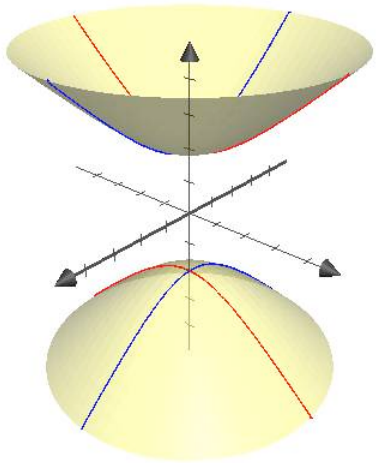
Forma cartesiana

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Forma Paramétrica

$x = a\cosh u\cos v$ donde:
$y = b\cosh usenv$ $u \in (0, 2\pi)$
$z = csenhu$ $v \in (0, \pi)$
a,b,c>0

4. HIPERBOLOIDE ELÍPTICO (DE 2 MANTOS -HOJAS-)



Forma cartesiana

$$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Forma Paramétrica

$$\begin{aligned} x &= asenhu\cos v \\ y &= bsenhusenv \\ z &= c\cosh v \end{aligned}$$

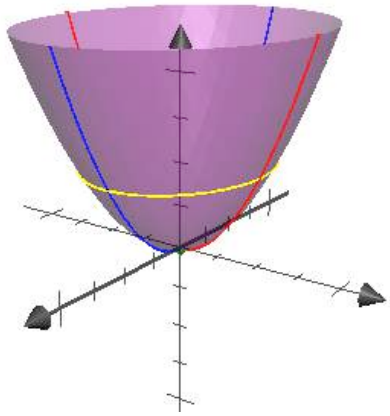
donde:

$u \in (0, 2\pi)$

$v \in (0, \pi)$

a,b,c>0

5. PARABOLOIDE ELÍPTICO



Forma cartesiana

$$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

Forma Paramétrica

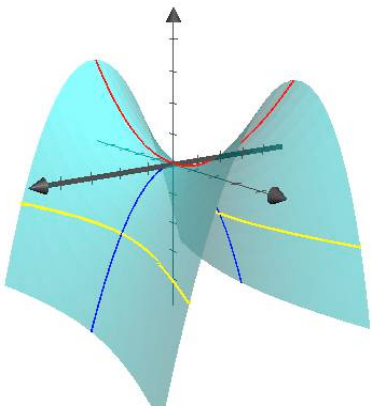
$$\begin{aligned} x &= a\ u\cos v \\ y &= b\ usenv \\ z &= c\ u^2 \end{aligned}$$

donde:

$v \in (0, 2\pi)$

a,b,c>0

6. PARABOLOIDE HIPERBÓLICO



Forma cartesiana

$$z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$$

Forma Paramétrica

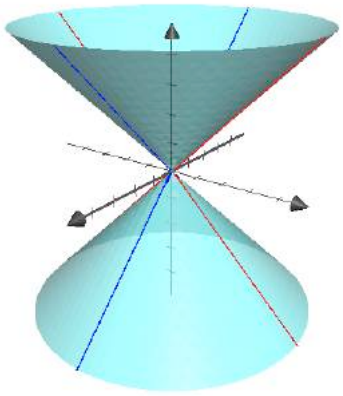
$$\begin{aligned} x &= a\ u, \\ y &= b\ v, \\ z &= u^2 - v^2, \end{aligned}$$

donde: $a,b>0$

$v \in (d,-d)$

$u \in (-c,c), c,d \in \Re$

7. CONO



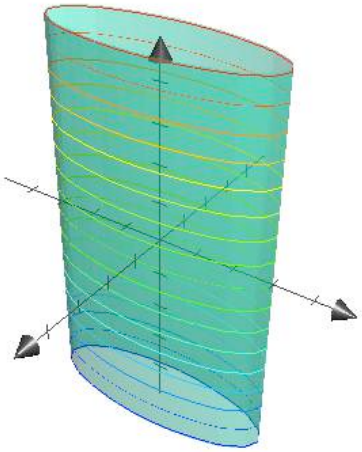
Forma cartesiana

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

Forma Paramétrica

$x = av\cos u$ donde:
$y = bvsenu$ $u \in (0, 2\pi)$
$z = cv$ $v \in \Re$
a,b,c>0

8. CILINDRO ELÍPTICO



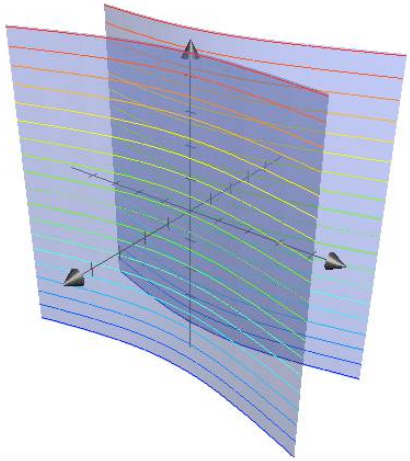
Forma cartesiana

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Forma Paramétrica

$x = a\cos u$ donde:
$y = bsenu$ $u \in (0, 2\pi)$
$z = v$ $v \in \Re$
a,b>0

9. CILINDRO HIPERBÓLICO



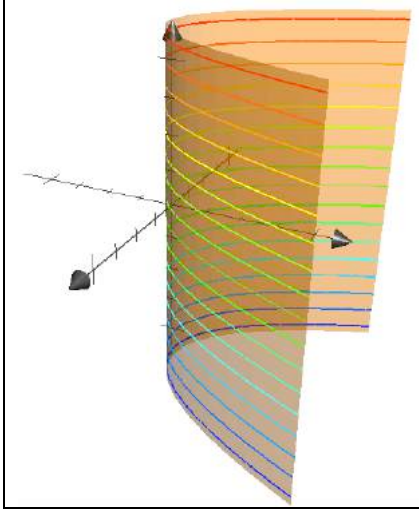
Forma cartesiana

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

Forma Paramétrica

$x = a\cosh u$ donde:
$y = bsenhu$ $u, v \in \Re$
$z = v$ a,b>0

10. CILINDRO PARABÓLICO



Forma cartesiana

$$\frac{x^2}{a^2} - 2py = 0$$

Forma Paramétrica

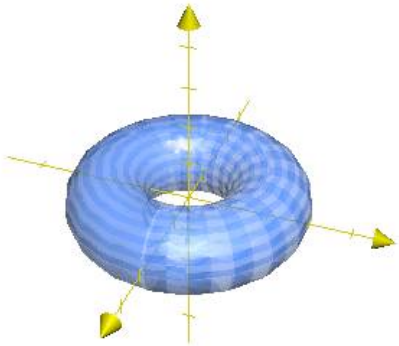
$$x = u$$
$$y = \frac{1}{2pu^2}$$
$$z = v$$

donde:

$v \in \Re$

$u, p \neq 0$

11. TORO (DONA)



Forma cartesiana

Forma Paramétrica

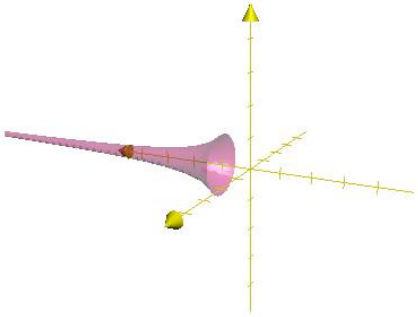
$$x = \cos u \left(a + b\cos v\right)$$
$$y = senu \left(a + b\cos v\right)$$
$$z = bsenv$$

donde:

$u,v \in \left(0, 2\pi\right)$

$a, b > 0$

12. TROMPETA DE GABRIEL



Forma cartesiana

Forma Paramétrica

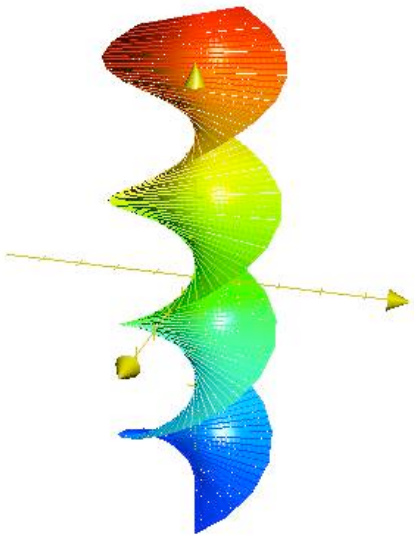
$$x = u$$
$$y = \frac{1}{u}\cos v$$
$$z = \frac{1}{u}senv$$

donde:

$v \in \left(0, 2\pi\right)$

$u \neq 0$

13. HELICOIDE

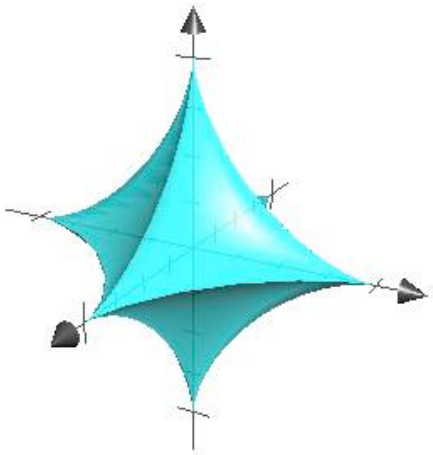


Forma cartesiana

Forma Paramétrica

$$x = u\cos v \qquad \text{donde:}$$
$$y = usenv \qquad v \in (0, 2\pi)$$
$$z = v \qquad u \in \Re$$

14. ESFERA ASTEROIDAL



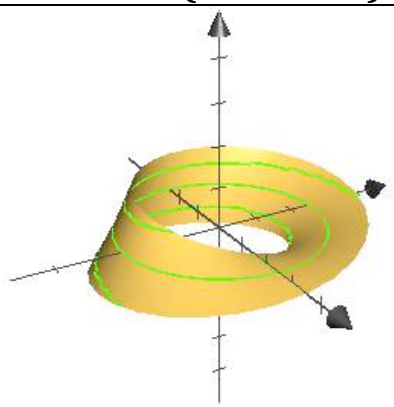
Forma cartesiana

$$x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}, a > 0$$

Forma Paramétrica

$$x = a(senu)^3(\cos v)^3 \qquad \text{donde: } a > 0$$
$$y = a(senu)^3(senv)^3 \qquad v \in (0, 2\pi)$$
$$z = a(\cos u)^3 \qquad u \in (0, \pi)$$

15. CINTA (O BANDA) DE MÖBIUS



Forma cartesiana

Forma Paramétrica

$$x = \left(a + u\cos\left(\frac{v}{2}\right)\right)\cos v, \qquad \text{donde: } a > 0$$
$$y = \left(a + u\cos\left(\frac{v}{2}\right)\right)senv, \qquad v \in (0, 2\pi)$$
$$z = usen\left(\frac{v}{2}\right) \qquad u \in (-c, c), c \in \Re$$